

QUANTITATIVE DUALITY AND QUANTUM ERASURE FOR NEUTRAL KAONS

A. Bramon, Grup de Física Teòrica, Universitat Autònoma de Barcelona,
E-08193 Bellaterra, Spain

G. Garbarino, Departament d'Estructura i Constituents de la Matèria, Universitat de Barcelona,
E-08028 Barcelona, Spain

B.C. Hiesmayr, Institute for Experimental Physics, University of Vienna,
A-1090 Vienna, Austria

Abstract

The aim of this contribution is to illustrate two basic aspects of quantum mechanics applied to the neutral kaon system. We first describe a recent quantitative formulation of Bohr's complementarity principle for free-space evolution of single kaons and entangled kaon pairs. We then show that the neutral kaon system is also suitable for an optimal demonstration of the "quantum eraser", including its operation in the "delayed choice" mode. In our discussions, strangeness oscillations play the role of the traditional interference pattern linked to wave-like behaviour. The role of the two interferometric paths taken by particle-like objects is played by the differently propagating K_S and K_L components. Their distinct decay widths provide a quantum "mark" which can be erased by appropriate strangeness measurements.

I. INTRODUCTION

As it is well known, fundamental aspects and principles of quantum mechanics can be conveniently illustrated by the basic double-slit experiment [1]. In order to do this, one usually explores the consequences of the complementarity relation existing between the observation of interference fringes, conventionally associated to wave-like behaviour, and the acquisition of "which way" or *Welcher Weg* information, similarly associated to particle-like behaviour.

In a double-slit-like experiment, interference patterns are observed if and only if it is impossible to know, *even in principle*, which path the particle took. Interference disappears if there is a way to know —e.g., through a *quantum marking* procedure— which path the particle took; whether or not the outcome of the corresponding "which way" observation is actually read out, it is completely irrelevant: information is there and interference is in any way lost. But, if that "which way" mark is erased by means of a suitable measurement —*quantum erasure* [2]—, interferences can be somehow restored.

The acquisition of "which way" information in an interferometric device is always accompanied by a reduction of the interference fringe visibility. This well known, *qualitative* observation has been recently translated into modern and *quantitative* expressions of Bohr's complementarity

[3, 4, 5, 6]. These quantitative complementarity expressions, as well as the quantum eraser effect, have been investigated and confirmed in a variety of recent interferometric experiments with atoms [6] or photon pairs [7].

The purpose of this contribution is to extend these considerations, quantitative complementarity (or duality) and quantum erasure, to neutral kaons. This is possible because a clear analogy exists between the neutral kaon propagation in free-space and the propagation of an interfering object in a double-slit experiment. Indeed, a neutral kaon beam presents the long known phenomenon of K^0 – \bar{K}^0 oscillations, which play the role of the interference fringes in a two-way interferometer experiment. Similarly, the short- and long-lived kaon states, K_S and K_L , being characterized by remarkably different decay widths ($\Gamma_S \gg \Gamma_L$) and distinct propagation in free-space, are the analogs of the two separated particle trajectories in interferometric devices. Therefore, the better one can know if the propagation proceeds through the K_S or K_L component —i.e., the more "*which width*" information one can have—, the less pronounced will be the K_S – K_L interference and the visibility of the K^0 – \bar{K}^0 oscillations.

For single neutral kaons we deduce a generalized version of the quantitative statement for complementarity proposed by Greenberger and Yasin [3] (Sec. III.A). Using instead entangled kaon pairs, another quantitative duality statement by Englert [4] and other authors [5, 6] is illustrated in Sec. III.B. Scully's quantum eraser [2] for the neutral kaon system is discussed in Sec. IV.

II. NEUTRAL KAONS

Two alternative bases, each one associated to a possible measurement, have to be considered for these neutral kaon analyses [10]. The strangeness basis, $\{K^0, \bar{K}^0\}$ with $\langle K^0 | \bar{K}^0 \rangle = 0$, is the appropriate one to discuss strong production and reactions of kaons, as well as kaon strangeness measurements. The latter are performed by inserting along the kaon trajectory a piece of ordinary matter which induces strangeness conserving kaon–nucleon strong interactions. The neutral kaon is then destroyed after being identified either as a K^0 or a \bar{K}^0 in a typically projective measurement.

The second basis consists of the K_S and K_L states hav-

ing well defined masses $m_{S(L)}$ and decay widths $\Gamma_{S(L)}$; it is the appropriate one to discuss neutral kaon propagation in free-space, with:

$$|K_{S(L)}(\tau)\rangle = e^{-i\lambda_{S(L)}\tau} |K_{S(L)}\rangle \quad (1)$$

and $\lambda_{S(L)} = m_{S(L)} - i\Gamma_{S(L)}/2$, as well as lifetime measurements. The K_S and K_L eigenstates do not oscillate into each other in time, but, since $\Gamma_S \simeq 579 \Gamma_L$, the short-lived component of a given neutral kaon extincts much faster than the long-lived one. Because of this lifetime “mark”, knowing if this kaon has propagated in free-space *either as K_S or K_L* is thus possible by detecting at which time it decays. If kaons decaying before $\simeq 4.8 \tau_S$ after production are identified as K_S ’s and those surviving after this time as K_L ’s, misidentifications amount to only a few parts in 10^{-3} [10].

The following relationship between the two kaon bases:

$$\begin{aligned} |K^0\rangle &= \frac{1}{\sqrt{2}} [|K_S\rangle + |K_L\rangle], \\ |\bar{K}^0\rangle &= \frac{2}{\sqrt{2}} [|K_S\rangle - |K_L\rangle], \end{aligned} \quad (2)$$

is valid when the small CP violating effects are neglected. Note that this is a good approximation for our purposes since these effects are of the same order as the previously mentioned K_S vs K_L misidentifications. Note also that the strangeness measurements and lifetime observations completely exclude each other: the former require the insertion of nucleonic matter, the latter propagation in free-space. Bohr’s complementarity principle is thus at work: if strangeness (lifetime) is known, both outcomes for lifetime (strangeness) are equally probable.

III. QUANTITATIVE COMPLEMENTARITY

A. Single kaons

Eqs. (1) and (2) imply that a kaon state which is produced at time $\tau = 0$ as a K^0 evolves in (proper) time according to the expression:

$$|K^0(\tau)\rangle = \frac{1}{\sqrt{2}} [e^{-i\lambda_S\tau} |K_S\rangle + e^{-i\lambda_L\tau} |K_L\rangle]. \quad (3)$$

By normalizing to kaons surviving up to time τ , the previous state can be more conveniently rewritten as:

$$\begin{aligned} |K^0(\tau)\rangle &= \frac{|K_S\rangle + e^{-\frac{1}{2}\Delta\Gamma\tau} e^{-i\Delta m\tau} |K_L\rangle}{\sqrt{1 + e^{-\Delta\Gamma\tau}}} \\ &\equiv \frac{1}{\sqrt{2}} [|K_S(\tau)\rangle + |K_L(\tau)\rangle] \end{aligned}$$

where $\Delta\Gamma \equiv \Gamma_L - \Gamma_S < 0$ and $\Delta m \equiv m_L - m_S$, or as:

$$\begin{aligned} |K^0(\tau)\rangle &= \frac{1 + e^{-\frac{1}{2}\Delta\Gamma\tau} e^{-i\Delta m\tau}}{\sqrt{2(1 + e^{-\Delta\Gamma\tau})}} |K^0\rangle \\ &+ \frac{1 - e^{-\frac{1}{2}\Delta\Gamma\tau} e^{-i\Delta m\tau}}{\sqrt{2(1 + e^{-\Delta\Gamma\tau})}} |\bar{K}^0\rangle, \end{aligned}$$

in the $\{K^0, \bar{K}^0\}$ basis.

One thus easily obtains the following τ -dependent transition probabilities:

$$|\langle K^0 | K^0(\tau) \rangle|^2 = \frac{1}{2} \{1 + \mathcal{V}_0(\tau) \cos(\Delta m \tau)\}, \quad (4)$$

$$|\langle \bar{K}^0 | K^0(\tau) \rangle|^2 = \frac{1}{2} \{1 - \mathcal{V}_0(\tau) \cos(\Delta m \tau)\}, \quad (5)$$

where:

$$\mathcal{V}_0(\tau) = \frac{1}{\cosh\left(\frac{\Delta\Gamma\tau}{2}\right)} \quad (6)$$

is the time-dependent fringe visibility of the \bar{K}^0 - K^0 oscillations. On the contrary, no K_S - K_L oscillations are expected:

$$\begin{aligned} |\langle K_L | K^0(\tau) \rangle|^2 &= \frac{1}{2} |\langle K_L | K_L(\tau) \rangle|^2 \\ &= \frac{1}{1 + e^{\Delta\Gamma\tau}}, \end{aligned} \quad (7)$$

$$\begin{aligned} |\langle K_S | K^0(\tau) \rangle|^2 &= \frac{1}{2} |\langle K_S | K_S(\tau) \rangle|^2 \\ &= \frac{1}{1 + e^{-\Delta\Gamma\tau}}. \end{aligned} \quad (8)$$

These observations admit the following interpretation, as discussed in [8, 9]. As soon as a K^0 is produced, it starts propagating in free-space in the coherent superposition of K_S and K_L [see Eq. (3)] and mimics the two-way propagation of any quantum system beyond a symmetrical double-slit. In the familiar double-slit case, the system follows the two paths without “jumping” from one to the other in the same way as K_S - K_L oscillations are “forbidden”. In the kaon case, however, there are not two separated trajectories but a single path comprising *automatically* (i.e., with no need of any double-slit like apparatus) the two components K_S and K_L . As previously stated, these two interfering components are marked by their different decay widths. But this is an intrinsic mark on each component which is *automatically* given by Nature.

For initial K^0 ’s surviving up to time τ , the probabilities for K_S and K_L propagation are

$$\begin{aligned} w_S(\tau) &\equiv |\langle K_S | K^0(\tau) \rangle|^2 = \frac{1}{1 + e^{-\Delta\Gamma\tau}}, \\ w_L(\tau) &\equiv |\langle K_L | K^0(\tau) \rangle|^2 = \frac{1}{1 + e^{\Delta\Gamma\tau}}. \end{aligned}$$

From these expressions one obtains the “width predictability” (corresponding to the “path predictability” defined in Ref.[3]):

$$\mathcal{P}(\tau) \equiv |w_S(\tau) - w_L(\tau)| = \left| \tanh\left(\frac{\Delta\Gamma\tau}{2}\right) \right|. \quad (9)$$

$\mathcal{P}(\tau)$ quantifies the a priori (i.e., before any measurement is performed) “which width” knowledge we have from the fact that a kaon which was created as a K^0 at time $\tau = 0$

has survived up to time τ . At $\tau = 0$, both components start propagating with the same probability, $w_S(0) = w_L(0) = 1/2$, and the “width predictability” vanishes, $\mathcal{P}(0) = 0$. In other words, there is no information on which component actually propagates and the visibility of strangeness oscillations is maximal, $\mathcal{V}_0(0) = 1$. However, since the K_S and K_L components are intrinsically “marked” by their different lifetimes, “which width” information is obtained for initial K^0 ’s surviving up to time τ and the corresponding oscillation visibility is thus reduced. The fringe visibility and “width predictability” of Eqs. (6) and (9) fulfill the equation:

$$\mathcal{V}_0^2(\tau) + \mathcal{P}^2(\tau) = 1, \quad (10)$$

which can be viewed as a generalization of the quantitative duality relation of Ref.[3] to situations where $\mathcal{P}(\tau)$ and $\mathcal{V}_0(\tau)$ are not constants.

Note that the same expressions of Eqs. (6) and (9) for $\mathcal{V}_0(\tau)$ and $\mathcal{P}(\tau)$ are valid when the kaon state produced at time $\tau = 0$ is a \bar{K}^0 . In this case, as well as in the previous one starting with an initial K^0 , the state remains quantum mechanically pure. The two terms in Eq. (10) add up to one rather than fulfilling the less stringent relation $\mathcal{V}_0^2 + \mathcal{P}^2 < 1$ for mixed states.

Recent measurements by the CPLEAR Collaboration [11] have been interpreted [9] in terms of the quantitative duality we have just discussed. The proton–antiproton annihilation processes, $p\bar{p} \rightarrow K^-\pi^+K^0$ or $p\bar{p} \rightarrow K^+\pi^-K^0$, were used to produce either a K^0 or a \bar{K}^0 initial state, which were then allowed to propagate in free-space. In a first experiment [12], strangeness oscillations were observed by detecting semileptonic neutral kaon decays. In another experiment [13], K^0 – \bar{K}^0 oscillations were observed via strangeness measurements monitored by kaon–nucleon strong interactions. The experimental data collected in both CPLEAR experiments are seen to fulfil the statement for quantitative duality of Eq. (10). Further experiments at the operating ϕ -factory Daphne [14], which copiously produces neutral kaons via strong $\phi \rightarrow K^0\bar{K}^0$ decays, could provide interesting and more accurate tests.

B. Entangled kaon pairs

The a priori knowledge on the kaon lifetimes and thus its time evolution, expressed in the previous section in terms of the predictability $\mathcal{P}(\tau)$, comes exclusively from knowing that the kaon remains undecayed at time τ . This knowledge can be obviously improved if a measurement is performed on the kaon state [4, 5, 6]. However, since strangeness measurements and lifetime observations are completely destructive and no other projective measurement is possible on neutral kaons, two-particle entanglement is necessary to achieve such a purpose. Working with entangled kaons, one can perform a measurement on one member (the *meter* kaon) which allows one to increase the information on the propagation mode of its partner (the *object* kaon) without this being annihilated.

To this aim, consider the decay of the ϕ -meson [14] (or, alternatively, S -wave $p\bar{p}$ annihilation [15]) into $K^0\bar{K}^0$ pairs. Just after the decay, i.e., at time $\tau = 0$, one has the following maximally entangled state:

$$\begin{aligned} |\phi(0)\rangle &= \frac{1}{\sqrt{2}}[|K^0\rangle_l|\bar{K}^0\rangle_r - |\bar{K}^0\rangle_l|K^0\rangle_r] \\ &= \frac{1}{\sqrt{2}}[|K_L\rangle_l|K_S\rangle_r - |K_S\rangle_l|K_L\rangle_r], \end{aligned}$$

where l and r denote the “left” and “right” kaon directions of motion. In the lifetime basis, the state evolution up to time τ_l (τ_r) along the left (right) beam is given by:

$$\begin{aligned} |\phi(\tau_l, \tau_r)\rangle &= \frac{1}{\sqrt{2}} \left\{ e^{-i(\lambda_L\tau_l + \lambda_S\tau_r)} |K_L\rangle_l |K_S\rangle_r \right. \\ &\quad \left. - e^{-i(\lambda_S\tau_l + \lambda_L\tau_r)} |K_S\rangle_l |K_L\rangle_r \right\}. \end{aligned}$$

It is convenient to consider only kaon pairs with both left and right members surviving up to τ_l and τ_r . These are described by the normalized state:

$$|\phi(\Delta\tau)\rangle = \frac{1}{\sqrt{1 + e^{\Delta\Gamma\Delta\tau}}} \left\{ |K_L\rangle_l |K_S\rangle_r - e^{i\Delta m\Delta\tau} e^{\frac{1}{2}\Delta\Gamma\Delta\tau} |K_S\rangle_l |K_L\rangle_r \right\}, \quad (11)$$

where $\Delta\tau \equiv \tau_l - \tau_r$. Eq. (11) can be rewritten as:

$$\begin{aligned} |\phi(\Delta\tau)\rangle &= \frac{1}{\sqrt{2(1 + e^{\Delta\Gamma\Delta\tau})}} \\ &\times \left\{ |K^0\rangle_l |K_S\rangle_r - |\bar{K}^0\rangle_l |K_S\rangle_r \right. \\ &\quad \left. - e^{i\Delta m\Delta\tau} e^{\frac{1}{2}\Delta\Gamma\Delta\tau} [|K^0\rangle_l |K_L\rangle_r + |\bar{K}^0\rangle_l |K_L\rangle_r] \right\}, \end{aligned} \quad (12)$$

or equivalently as:

$$\begin{aligned} |\phi(\Delta\tau)\rangle &= \frac{1}{2\sqrt{1 + e^{\Delta\Gamma\Delta\tau}}} \\ &\times \left\{ \left(1 - e^{i\Delta m\Delta\tau} e^{\frac{1}{2}\Delta\Gamma\Delta\tau} \right) \right. \\ &\quad \times [|K^0\rangle_l |K^0\rangle_r - |\bar{K}^0\rangle_l |\bar{K}^0\rangle_r] \\ &\quad \left. + \left(1 + e^{i\Delta m\Delta\tau} e^{\frac{1}{2}\Delta\Gamma\Delta\tau} \right) \right. \\ &\quad \left. \times [|K^0\rangle_l |\bar{K}^0\rangle_r - |\bar{K}^0\rangle_l |K^0\rangle_r] \right\}. \end{aligned} \quad (13)$$

Eqs. (11)–(13) immediately supply the various joint probabilities $P(K_l, K_r)$ for detecting a K_l (K_r) on the left (right) at time τ_l (τ_r) with $K_{l,r} = K^0, \bar{K}^0, K_S$ or K_L .

Let us now consider the following two experimental arrangements [8]. In both arrangements one measures the strangeness of the left moving kaon by inserting a dense slab of (nucleonic) matter at different distances along its trajectory; since each distance corresponds to a given time-of-flight τ_l , one can measure at different values of τ_l and

look for strangeness oscillations of this (object) kaon. On the other hand, the measurement on the right moving (meter) kaon is always performed at a *fixed* time τ_r^0 ; but on this kaon, one of two alternative measurements, $\mathcal{M} = \mathcal{S}$ or \mathcal{L} , is performed by either inserting or not a strangeness detector at τ_r^0 . In the first case, $\mathcal{M} = \mathcal{S}$, strangeness is measured on both kaons and four τ_l -dependent joint probabilities $P(K_l, K_r)$ with $K_{l,r} = K^0$ or \bar{K}^0 can be recorded. In the second case, by removing the strangeness detector one observes the lifetime of the right moving kaon, $\mathcal{M} = \mathcal{L}$, and measures the four joint probabilities $P(K_l, K_r)$ with $K_l = K^0$ or \bar{K}^0 and $K_r = K_S$ or K_L (for explicit expressions and details see Sec.IV.).

Following Englert [4], we can then define the “which width knowledge” for the object kaon, $\mathcal{K}_{\mathcal{M}}(\tau_l)$, which depends on the measurement (either $\mathcal{M} = \mathcal{S}$ or $\mathcal{M} = \mathcal{L}$) performed on the meter kaon. Note however that an experiment can never decrease the information provided by the formerly discussed predictability $\mathcal{P}(\tau_l)$, and one necessarily has $\mathcal{K}_{\mathcal{M}}(\tau_l) \geq \mathcal{P}(\tau_l) \equiv |\tanh(\Delta\Gamma \tau_l/2)|$. One can similarly consider the corresponding visibility of the object kaon strangeness oscillations, $\mathcal{V}_{\mathcal{M}}(\tau_l)$, which now satisfies $\mathcal{V}_{\mathcal{M}}(\tau_l) \leq \mathcal{V}_0^2(\tau_l) \equiv 1/\cosh(\Delta\Gamma \tau_l/2)$. It will be seen that $\mathcal{K}_{\mathcal{M}}(\tau_l)$ and $\mathcal{V}_{\mathcal{M}}(\tau_l)$ are linked by a new quantitative duality relation [5, 6]:

$$\mathcal{V}_{\mathcal{M}}^2(\tau_l) + \mathcal{K}_{\mathcal{M}}^2(\tau_l) = 1. \quad (14)$$

Without the strangeness detector on the right beam, $\mathcal{M} = \mathcal{L}$, one can observe the decay of the freely propagating meter kaon, which will be identified either as K_S or K_L . The acquisition of this “which width” information implies the corresponding one for the object kaon and therefore strangeness oscillations (in τ_l) should not be visible:

$$\mathcal{V}_{\mathcal{L}}(\tau_l) = 0 \quad \forall \tau_l, \quad (15)$$

for any of the four possible joint detection probabilities $P(K_l, K_r)$ with $K_l = K^0$ or \bar{K}^0 and $K_r = K_S$ or K_L . This is immediately seen from Eq. (12). Using Eq. (11) one obtains the maximum value for the object kaon “which width knowledge” [4]:

$$\mathcal{K}_{\mathcal{L}}(\tau_l) \equiv |P(K_S(\tau_l), K_S(\tau_r^0)) - P(K_L(\tau_l), K_S(\tau_r^0))| + |P(K_S(\tau_l), K_L(\tau_r^0)) - P(K_L(\tau_l), K_L(\tau_r^0))| = 1 \quad \forall \tau_l,$$

where now the joint probabilities explicitly show their dependence on left and right measurement times τ_l and τ_r^0 . The quantitative duality relation of Eq. (14) is then trivially satisfied.

Considering instead a strangeness measurement along the right beam, $\mathcal{M} = \mathcal{S}$, one obtains the four joint probabilities $P(K_l, K_r)$ with $K_{l,r} = K^0$ or \bar{K}^0 which show the τ_l -dependent strangeness oscillations and anti-oscillations immediately deducible from Eq. (13), with visibility:

$$\mathcal{V}_{\mathcal{S}}(\tau_l) = \frac{1}{\cosh\left(\frac{\Delta\Gamma(\tau_l - \tau_r^0)}{2}\right)}. \quad (16)$$

This visibility is maximal ($\mathcal{V}_{\mathcal{S}} = 1$) for $\tau_l = \tau_r^0$ (no “which width” information available), but the strangeness oscillations disappear ($\mathcal{V}_{\mathcal{S}} \rightarrow 0$) for $\tau_l \rightarrow \infty$ (full “which width” information). The “which width” information on the object kaon, which is available as a result of the joint strangeness measurement, can be expressed in terms of the “which width knowledge” [4]:

$$\begin{aligned} \mathcal{K}_{\mathcal{S}}(\tau_l) &\equiv |P[K_S(\tau_l), K^0(\tau_r^0)] - P[K_L(\tau_l), K^0(\tau_r^0)]| \\ &\quad + |P[K_S(\tau_l), \bar{K}^0(\tau_r^0)] - P[K_L(\tau_l), \bar{K}^0(\tau_r^0)]| \\ &= \left| \tanh\left(\frac{\Delta\Gamma(\tau_l - \tau_r^0)}{2}\right) \right|, \end{aligned}$$

which, together with the visibility (16), satisfies again the quantitative duality requirement (14).

Note that the two types of alternative measurements performed on the right moving kaon are clearly different. If strangeness is measured, $\mathcal{M} = \mathcal{S}$, no “which width” information is obtained in addition to the information that was already known a priori, thus $\mathcal{K}_{\mathcal{S}}(\tau_l) \equiv \mathcal{P}(\tau_l)$. In the other case, $\mathcal{M} = \mathcal{L}$, the opposite situation is achieved and the “which width” knowledge is maximally increased, $\mathcal{K}_{\mathcal{L}}(\tau_l) \equiv 1 > \mathcal{P}(\tau_l)$. According to the definition of Ref. [4], this corresponds to a “width distinguishability” $\mathcal{D}(\tau_l) \equiv 1$.

IV. QUANTUM ERASER

The discussion of Sec.III. can be reinterpreted in terms of Scully’s *quantum eraser* [2], which consists of three different steps [16].

In the first step one has to start with a coherent kaon superposition of K_S and K_L states such as the one in Eq. (3), which provides K_S - K_L interference effects showing the K^0 - \bar{K}^0 oscillations of Eqs. (4) and (5). These oscillations become less pronounced with time as a consequence of the increase of the K_S and K_L “path predictability”.

In the second step one considers entangled kaon pairs in the state (11). If the right going meter kaon is free to propagate in space, its lifetime “mark” is operative and precludes the observation of any strangeness oscillation of the entangled partner, the object kaon. The following joint probabilities are then measured:

$$\begin{aligned} P[K^0(\tau_l), K_S(\tau_r^0)] &= P[\bar{K}^0(\tau_l), K_S(\tau_r^0)] \quad (17) \\ &= \frac{1}{2(1 + e^{\Delta\Gamma(\tau_l - \tau_r^0)})}, \\ P[K^0(\tau_l), K_L(\tau_r^0)] &= P[\bar{K}^0(\tau_l), K_L(\tau_r^0)] \\ &= \frac{1}{2(1 + e^{-\Delta\Gamma(\tau_l - \tau_r^0)})}. \end{aligned}$$

The possibility to obtain full “which width” information for the object kaon can be precluded by the third step of the quantum eraser. In order to erase the lifetime “mark” of the meter kaon, it has to be measured in the $\{K^0, \bar{K}^0\}$ basis of Eqs. (2); strangeness oscillations and their complementary

anti-oscillations appear when sorting the jointly detected events according to the following probabilities:

$$\begin{aligned}
P[K^0(\tau_l), K^0(\tau_r^0)] &= P[\bar{K}^0(\tau_l), \bar{K}^0(\tau_r^0)] \quad (18) \\
&= \frac{1 - \mathcal{V}(\tau_l) \cos[\Delta m(\tau_l - \tau_r^0)]}{4}, \\
P[K^0(\tau_l), \bar{K}^0(\tau_r^0)] &= P[\bar{K}^0(\tau_l), K^0(\tau_r^0)] \\
&= \frac{1 + \mathcal{V}(\tau_l) \cos[\Delta m(\tau_l - \tau_r^0)]}{4}.
\end{aligned}$$

Note that this third step restores the same K_S – K_L interference effects [see Eqs.(4) and (5)] of the first step, where single kaon states [see Eq. (3)] are used.

Note also that the joint probabilities of Eqs. (17) and (18), $\mathcal{K}_{\mathcal{M}}$ and $\mathcal{V}_{\mathcal{M}}$ are all of them even functions of $\tau_l - \tau_r^0$. The erasure of the lifetime “mark” on the meter kaon can thus be delayed to times after the object kaon has been detected ($\tau_r^0 > \tau_l$). This “delayed choice” mode captures the essential feature of the quantum eraser [2], which is a proper sorting of the jointly detected events.

V. CONCLUSIONS

To summarize, we have discussed new quantitative formulations of Bohr’s complementarity for free-space propagation of single and entangled pairs of neutral kaons. Single neutral kaons allow for a generalization and clear application of the Greenberger and Yasin duality relation [Eq. (10)]. In this case, the two recent CPLEAR experiments of Refs. [12, 13] admit a transparent interpretation in terms of the quantitative complementarity requirement of Eq. (10) and the data fully agree with this equality. Entangled $K^0\bar{K}^0$ pairs allows for the more interesting complementarity test in terms of “width knowledge” and “width distinguishability” suggested by Englert and other authors. Experimental tests on this issue could be performed at the ϕ -factory Daphne [14].

The neutral kaon system reveals also suitable for an optimal demonstration [2, 16] of quantum erasure: (1) the “which width” information is carried by a system (the meter kaon) distinct and spatially separated from the interfering system (the object kaon); (2) as a consequence, the erasure operation can be performed in the “delayed choice” mode ($\tau_l < \tau_r^0$); (3) single-particle states (as opposed to coherent states) are detected on each side; (4) in spite of the need to entangle the object kaon with another system (the meter kaon), quantum erasure allows one to restore the same K_S – K_L interference phenomenon exhibited by a single kaon state produced as K^0 or \bar{K}^0 .

An experimental test of the marking and erasure operations we have discussed is desirable and should be feasible at ϕ -factories and $p\bar{p}$ machines. Actually, the CPLEAR collaboration [15] has already done part of the work required: the two experimental points (for $|\tau_l - \tau_r^0| = 0$ and $1.2\tau_S$) collected by this experiment are compatible with the joint strangeness oscillations predicted by Eqs. (18). New measurements confirming with better precision these

oscillations for a larger range of $\tau_l - \tau_r^0$ values, as well as the non-oscillating behaviour when “which width” information is in principle available [see Eq. (17)], are needed for a full demonstration of quantitative duality and the quantum eraser with neutral kaons.

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